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MEASURING RETURNS TO SCALE BASED ON THE TRIANGULAR FUZZY DEA APPROACH WITH DIFFERENT VIEWS OF EXPERTS: CASE STUDY OF IRANIAN INSURANCE COMPANIES

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Abstract: *The importance of insurance companies in the economic growth of countries has led to them, so in this article, the efficiency of insurance companies is measured based on inputs, favorable and unfavorable outputs. The developed model, unlike the previous models, considers the unfavorable outputs of insurance companies in conditions of uncertainty with fuzzy data based on different views of experts. The required data for each of the inputs and outputs have been provided by experts in the form of triangular fuzzy numbers. The existence of different views of experts, including optimistic, likely, and pessimistic, has led to its impact on the returns to the scale of insurance companies. The results of the survey of 24 insurance companies in Iran, based on the different views of experts, show that the more optimistic the experts' view is, the higher the average return on the scale of insurance companies compared to other views. As the expert view has shifted from optimistic to pessimistic, returns to full scale for insurance companies have declined. In this way, the average return to the scale of all insurance companies is equal to 0.8972 in the optimistic view, in the probable view it is equal to 0.8863 and in the pessimistic view it is equal to 0.8336. The uncertainty rate also affects the inputs, desirable and undesirable outputs of the model, and with the increase of this rate, the desirable inputs and outputs decrease and the undesirable outputs increase. The result of this is the reduction of the average return to the scale of insurance companies with the increase of the uncertainty rate.*

Key words*:* *Fuzzy data envelopment analysis, uncertainty rate, insurance companies, returns to scale.*

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1. Introduction

Insurance has been proposed as a financial concept for risk management, which seeks to create security for economic growth or protection against possible losses. The insurance industry is always full of challenges, unlike the market growth trend (Sinha, 2019). Insurance companies compete and operate in a rapidly changing environment that requires objective and reliable measurement of individual performance and effectiveness. As long as insurance companies cannot provide the necessary platform for the safe presence of investors in various economic sectors, economic growth cannot be expected in that country (Nourani et al., 2018). Therefore, inefficiency in the insurance industry not only leads to a low quality of life, but also hinders the improvement of efficiency in the economic sectors and, as a result, their growth. Therefore, in order to survive and compete in a dynamic environment, the insurance industry needs to evaluate the correct performance and, if necessary, improve efficiency. Meanwhile, various approaches have been developed to evaluate and compare the development of the performance of insurance companies, which can be referred to the method of data coverage analysis (Suvvari et al., 2019). Data envelopment analysis has been used as a non-parametric approach to identify efficient centers. This quantitative method has been used to measure efficiency to the relative technical scale of organizational units. It is a technique based on mathematical optimization that can maximize outputs according to specified inputs or minimize inputs to produce a fixed output (Abd Aziz et al., 2022; Ucal Sari & Ak, 2022).

By identifying the efficiency of insurance companies based on inputs and outputs, it is possible to attract domestic and foreign investors with more confidence, which can lead to prosperity and economic growth. In today's turbulent environment, it is difficult to accurately determine the amount of inputs and outputs of any industry, including insurance companies (Shahroudi et al., 2011). Because the uncertainty of the happenings for insurance companies denies the possibility of accurate calculation of profit, total cost, company debts, losses, etc. (Jaloudi, 2019). Therefore, control methods such as mathematical planning based on experts' opinions should be used in calculating returns to the scale of insurance companies. In this method, according to the opinions of experts, each of the required inputs and outputs is categorized into three levels: optimistic, probable and pessimistic. The opinions of each of the experts with different views can overshadow the scale efficiency of each of the insurance companies. Therefore, a more comprehensive model should be implemented to deal with such cases. The presence of uncertainty rate in the fuzzy programming method (Farnam & Darehmiraki, 2022; Mekawy, 2022; Das, 2022; Farnam & Darehmiraki, 2021) can control the different views of experts, including pessimistic, likely and optimistic, and ensure the obtained results. Also, in the calculation of returns to the scale of insurance companies, only the desired outputs are considered. Because insurance companies are not measured only by the favorable output, and there are also unfavorable outputs in the evaluation of their return to scale.

In this article, according to the stated cases, a new model of data envelopment analysis (CCR model) based on the simultaneous use of favorable and unfavorable outputs in conditions of uncertainty is presented. In this model, each of the experts can have different optimistic, probable and pessimistic views towards input and output fuzzy data. Therefore, the presented comprehensive model has the ability to examine the returns to the scale of insurance companies in uncertain conditions of inputs, favorable and unfavorable outputs. In order to examine the returns to the scale of the model in the real world, 24 active insurance companies in Iran were selected and returned to their scale in terms of inputs (total assets, share capital, fees paid), desired output (net profit, reinsurance reserves, health and life, the number of damages paid) and adverse output (company debts, dissatisfaction and losses incurred) have been measured in different optimistic, probable and pessimistic perspectives. Therefore, the contribution of the article is as follows:

- Development of data envelopment analysis model in fuzzy conditions.
- The simultaneous use of favorable and unfavorable outputs in measuring returns to the scale of insurance companies.
- Considering the optimistic, probable, and pessimistic views of experts in the discussion of measuring the return to the scale of insurance companies.
- Combining qualitative and quantitative indicators in the mathematical model

The structure of the article is as follows: in the second part, the literature review is discussed and the research gap is identified. In the third part, the modeling of yield measurement to the scale of insurance companies based on the data coverage analysis model under uncertainty conditions has been discussed. In this section, the fuzzy programming method with α cut is used to control inputs, and desirable and undesirable outputs. In the fourth part, the implementation of the mathematical model in insurance companies in Iran has been discussed. Finally, in the fifth section, conclusions and suggestions for future research have been discussed.

2. Literature Review

Many articles have dealt with the measurement of efficiency at the scale of industrial units, public and private companies. Most of the studied articles have used output-based models. Barros and Wanke (2014) investigated the efficiency of Mozambican insurance companies using Hief data envelopment analysis. The results show that the production growth potential of Mozambican insurance companies is severely limited, especially in terms of the growth potential of ceded reinsurance. Micajkova (2015) investigated the return to scale of the insurance sector of the Republic of Macedonia during the period 2013-2009. They measured the technical, net technical and scale efficiency of 11 Macedonian insurance companies using data envelopment analysis, both CCR and BCC models. Kaffash et al. (2020) reviewed 620 articles published in journals indexed in the Web of Science database from 1985 to April 2016 in the development and application of data envelopment analysis. Bao et al. (2018) evaluated insurance performance. Locally in Malaysia for the period 2014-2015. They used data envelopment analysis based on CCR to measure returns to scale. They used three inputs and three outputs, including operating, capital and commission costs, as well as net premiums, net investment income and net claims incurred. The results showed that there are 8 efficient companies in 2014 and 9 efficient companies in 2015. The average return-to-scale score increased from 78.9% in 2014 to 79.1% in 2015. Gharakhani et al. (2018) proposed an objective programming approach to generate joint weights in dynamic DEA network. To validate the applicability of the proposed model, they used the data of 30 Iranian non-life insurance companies during 2013-2014 to measure the efficiency score and rank all companies. Grmanová and Pukala (2018) compared the return to scale of life insurance of commercial insurance companies in the Czech Republic and Poland using data envelopment analysis model. They selected 17 commercial insurance companies in the Czech Republic and 26 commercial insurance companies in Poland. The results showed that the arithmetic mean of performance scores in Poland is higher than in the Czech Republic. Also, the diversity of efficiency scores of Czech insurance companies was more than the diversity of efficiency scores of Polish insurance companies.

Peykani et al. (2021) presented a new approach to measuring returns to scale and rank stocks. They used fuzzy data envelopment analysis with a probabilistic domain to measure stock returns. Using data from the insurance industry, this model was implemented for a real case study of the Tehran Stock Exchange in order to analyze the performance of the proposed method. Li et al. (2020) measured the operational efficiency of China's property insurance industry using data envelopment analysis. They selected 44 Chinese property insurance companies as the research target. The empirical results in the first stage show that the operation efficiency of China's property insurance industry is technically inefficient and scale efficiency is relatively better than pure technical efficiency. In the second stage, it was observed that the drivers of company size, reinsurance rate, loss ratio, and equity limit are important determinants of insurance company efficiency.

Naushad et al. (2020) used data envelopment analysis to calculate the managerial efficiency of 30 insurance companies listed on the Saudi Stock Exchange for a period of four years from 2015 to 2018. They considered two inputs (general and administrative costs and policy and acquisition costs), and two outputs (net premiums and investment income and other income) for efficiency calculations. The final results of this study show that a large number of insurance companies operating in KSA are efficient in the management efficiency scale.

Kaffash et al. (2020) conducted a case study regarding the use of data envelopment analysis to measure the efficiency of insurance companies during the years 1993 to 2018. In this article, they categorized the inputs and outputs of insurance companies in measuring their efficiency. Peykani et al. (2021) presented a new approach for performance evaluation and ranking of decision-making units with a two-stage network structure in the presence of imprecise and ambiguous data. In order to achieve this goal, the two-stage data envelopment analysis model, adjustable probabilistic programming, and chance-constrained programming were used to propose a new approach for fuzzy network data envelopment analysis.

Ghosh et al. (2021) investigated the performance of occupational life insurance companies in India during the period 2010 to 2017. He used the input-oriented data coverage analysis model to measure returns to the scale of insurance companies. It also performed a comparative analysis with other multi-criteria decision-making techniques, such as simple additive weighting, product-weighted summation evaluation, and weighted summation method. Zhao et al. (2021) investigated the evolution and determinants of profitability of 53 Chinese insurers during 2017-2013 using the data coverage analysis method. Also, they used Tobit regression models to examine several factors affecting profitability. The empirical results show the importance of the proper arrangement of costs and revenues for an insurer and help to better understand the effect of company size, age, and product characteristics on profitability. Omrani et al. (2022) introduced a data envelopment analysis method to measure the efficiency to scale of a two-step process. In this model, they used the ideal programming method in order to combine the objective functions of the problem. To check the validity of their model, they measured the return to scale of 22 insurance companies to determine the return to scale of insurance companies in Iran. Puspitasari and Fauziyah (2022) analyzed the level of efficiency of Sharia public insurance in Indonesia. They used data envelopment analysis method with output orientation. The purpose of this study was to 8 general insurance companies in Indonesia during the period of 2015-2020. The variables used to consist of input variables (total assets, capital/equity, operating expenses, payment of receivables) and output variables (investment income, axed funds, profit). Abdin et al. (2022) analyzed the efficiency of Indonesian public insurance companies using two stages of data envelopment analysis during 2017-2018. The first stage of efficiency measurement using the non-parametric data coverage analysis approach shows the efficiency level of public insurance companies that experience a positive trend. In the second step, using the Tobit regression model, it was shown that the cost ratio is the only factor that significantly affects the level of return to scale of public insurance companies in Indonesia.

Uckar & Petrovic (2022) compared the returns to scale of Croatian insurance companies using traditional financial indicators and non-parametric DEA methodology in the period from 2015 to 2020. The results show that the average return to scale of insurance companies has improved in the observed period, while the gap between large, medium and small insurers continues to increase. Shobeiri et al. (2022) used data envelopment analysis to provide a model for predicting the risk of insurers (in terms of the presence of risk of loss or the absence of risk of loss). Their model was implemented on the data of car insurance policies of Saman Insurance Company during the years 2018-2019. Omari et al. (2023) investigated the insurance companies' financial performance in Jordan's Amman Stock Exchange (ASE). The sample size is 15 out of 22 selected insurance firms from 2008 to 2020. The results show that MLP is efficient and performs well in multiple criterion tests through iteration growth. Raj et al. (2023) developed a three-stage closed system FDEA to assess the competency of reinsurers functioning in India. The FDEA findings reveal the individual efficiency score of production, investment, and the effectiveness sub-processes of each Decision-Making Unit. Ashiagbor et al. (2023) employed the non-parametric Malmquist productivity change indices and the bootstrap technique to measure the productivity changes of a sample of 19 out of 20 Ghanaian life insurance firms for the period 2015–2020. The empirical results show that the level of output-oriented technical inefficiency of the life insurance industry in Ghana is approximately 17% over the period 2015–2020. Sadeghi et al. (2023) investigated the dual effect of marketing and profit creation in insurance companies has using DEA approach. The results show that in the three periods studied, Asia, Parsian, Dey, Pasargad, Kowsar and Ta'avon insurance company were fully efficient and Novin Insurance Company had the lowest efficiency.

In Table 1, the research gap of the study has been examined.

Table 1. The research gap of study

Reff	Purpose	Deterministic/ Uncertainty	Solution Method	Favorable and Unfavorable Index	Expert Opinion	Case Study
Li et al. (2020)	Efficiency of China's property insurance industry	D	DEA			China's property insurance industry
Peykani et al. (2021)	performance evaluation and ranking of decision-making units	F	FDEA		*	

Reff	Purpose	Deterministic/ Uncertainty	Solution Method	Favorable and Unfavorable Index	Expert Opinion	Case Study
Uckar & Petrovic (2022)	Compared the returns to scale of Croatian insurance companies	D	DEA			Croatian insurance companies
Omrani et al. (2022)	Measure the efficiency to scale of a two-step process	D	DEA			Insurance companies in iran
Raj et al. (2023)	Investigating the efficiency of insurance companies	U	FDEA		*	Competency of reinsurers functioning in india
Omari et al. (2023)	Investigated the insurance companies' financial performance	D	DEA	-	-	Jordan's amman stock exchange
This paper	Investigating the efficiency of insurance companies	U	FDEA	*	*	Iranian insurance companies

The results of the literature review show that there are various papers in the field of measuring the efficiency of insurance companies in the world using the method of data envelopment analysis. But by studying the literature review, it can be stated that there are very few models using the fuzzy programming method with optimistic, probable and pessimistic views to measure the return on the scale of insurance companies. Also, in these analyses, the outputs of the problem have only been considered as desirable outputs, which is necessary to develop the model to consider both desirable and undesirable outputs.

3. Problem Definition and Modeling

In this part of the paper, a data envelopment analysis model has been presented as a mathematical programming method to measure efficiency to the scale of DMUs. In this model, assume that there are N decision-making units ($DMU_n,\ n \in \{1,2,\dots,N\}$). Each DMU_n has I input ($\tilde{a}_{in}, i \in \{1,2,\dots,I\}$)، J desirable output $\left(\tilde{b}_{jn}, j \in \{1,2,\dots,J\}\right)$ and K undesirable output ($\tilde{c}_{kn}, k \in \{1,2,\dots,K\}$). This means that the inputs to the problem have the possibility of creating both desirable and undesirable outputs at the same time. Also, DMU_0 as the basic decision-making unit, v_j is the weighting coefficient of the desirable output, w_k is the weighting coefficient of the undesirable output, and u_i

is the weighting coefficient of the problem inputs, which is determined by the model. Based on the development of the basic model of CCR data envelopment analysis (Charnes et al., 1978), the nonlinear mathematical programming model under uncertainty is as follows:

$$Max\ \theta_0 = \frac{\sum_{j=1}^{J} v_j \tilde{b}_{j0} - \sum_{k=1}^{K} w_k \tilde{c}_{k0}}{\sum_{i=1}^{I} u_i \tilde{a}_{j0}} \tag{1}$$

$$s.t:$$

$$0 \leq \frac{\sum_{j=1}^{J} v_j \tilde{b}_{jn} - \sum_{k=1}^{K} w_k \tilde{c}_{kn}}{\sum_{i=1}^{I} u_i \tilde{a}_{jn}} \leq 1, \quad \forall n \in N \tag{2}$$

$$v_j, w_k, u_i \geq 0 \tag{3}$$

Eq. (1) deals with the measurement of efficiency to the scale of the basic decision-making unit based on the outputs of the decision-making unit (desirable and undesirable) on the inputs of the decision-making unit. Eq. (2) also shows that the return to scale of each decision-making unit must be between 0 and 1. Eq. (3) also shows that the weight coefficients of inputs and outputs must be a positive number. Considering that the presented model is a non-linear mathematical programming model, it can be converted into a linear mathematical programming model using the following relationships.

$$Max\ \theta_0 = \sum_{j=1}^{J} v_j \tilde{b}_{j0} - \sum_{k=1}^{K} w_k \tilde{c}_{k0} \tag{4}$$

$$s.t:$$

$$\sum_{i=1}^{I} u_i \tilde{a}_{j0} = 1 \tag{5}$$

$$\sum_{j=1}^{J} v_j \tilde{b}_{jn} - \sum_{k=1}^{K} w_k \tilde{c}_{kn} \leq \sum_{i=1}^{I} u_i \tilde{a}_{jn}, \quad \forall n \in N \tag{6}$$

$$\sum_{j=1}^{J} v_j \tilde{b}_{jn} - \sum_{k=1}^{K} w_k \tilde{c}_{kn} \geq 0, \quad \forall n \in N \tag{7}$$

$$v_j, w_k, u_i \geq 0 \tag{8}$$

In the above model, it can be seen that the inputs, desirable and undesirable outputs are considered non-deterministic. Due to the lack of historical data in the inputs and outputs of the model and the use of experts' opinions, the fuzzy programming method should be used. In the following, the fuzzy programming method is discussed. Fuzzy programming method is used in order to control the confidence level of establishing non-deterministic limits as a suitable safe margin for establishing each of the limits. To do this, two standard fuzzy method measures, named fuzzy optimistic (POS) and fuzzy pessimistic (NEC), are commonly used. It is worth mentioning that the optimistic fuzzy represents the level of optimistic probability of the occurrence of an uncertain event including uncertain parameters, while the pessimistic fuzzy represents a pessimistic decision about the uncertain event. In this paper, pessimistic-optimistic hybrid fuzzy is used to control the inputs and outputs of the CCR model, That is, it is assumed that decision-making has a pessimistic and optimistic attitude at the same time to establish uncertain limits. Now, based on the aforementioned fuzzy parameters and using the expected value for the objective function and the pessimistic-optimistic action for the non-deterministic constraints, an axiomatic equivalent of the original non-deterministic model can be formulated. Assume that $\tilde{c}$ is a triangular fuzzy number. As a result, the membership function of this fuzzy number is defined as relation (9):

$$\mu\tilde{C}(x) = \begin{cases} f_c(x) = \dfrac{x - c_p}{c_m - c_p} & if\ c_p \leq x \leq c_m \\ 1 & if\ x = c_m \\ g_c(x) = \dfrac{c_o - x}{c_o - c_m} & if\ c_m \leq x \leq c_o \\ 0 & if\ x < c_p\ or\ x > c_o \end{cases} \tag{9}$$

The expected distance EI and mathematical expectation EV are calculated from the triangular fuzzy number from the following relations:

$$EI(\tilde{C}) = [E_1^c, E_2^c] = \left[\int_0^1 f_c^{-1}(x)dx\,, \int_0^1 g_c^{-1}(x)dx\right] = \left[\frac{1}{2}(c_m + c_p), \frac{1}{2}(c_o + c_m)\right] \tag{10}$$

$$EV(\tilde{C}) = \frac{E_1^c + E_2^c}{2} = \frac{c_p + 2c_m + c_o}{4} \tag{11}$$

For two fuzzy numbers $\tilde{a}$ and $\tilde{b}$, the degree of $\tilde{a}$ being greater than $\tilde{b}$ is defined as follows:

$$\mu_M(\tilde{a}, \tilde{b}) = \begin{cases} 1 & if\ E_1^a > E_2^b \\ \dfrac{E_2^a - E_1^b}{E_2^a - E_1^b - (E_1^a - E_2^b)} & if 0 \epsilon [E_1^a - E_2^b, E_2^a - E_1^b] \\ 0 & if\ E_2^a < E_1^b \end{cases} \tag{12}$$

As a result, when $\tilde{a}$ is greater than $\tilde{b}$ we have:

$$\frac{E_2^{a_i x} - E_1^{b_i}}{E_2^{a_i x} + E_1^{b_i} - E_1^{a_i x} - E_2^{b_i}} \geq \alpha, \forall i = 1, \dots, l \tag{13}$$

In other words, the simplified relationship above shows:

$$x \geq (1 - \alpha)E_1^{b_i} + \alpha E_2^{b_i}, \forall i = 1, \dots, l$$

The above relationship is when the decision maker approaches the problem pessimistically. The change in the above relations for decision making in optimistic conditions is as follows:

$$x \geq (1 - \alpha)E_2^{b_i} + \alpha E_1^{b_i}, \forall i = 1, \dots, l \tag{14}$$

In other words, when the decision maker makes a decision simultaneously, the data of the problem is controlled in the optimistic, pessimistic fuzzy method as described in the following relationship:

$$x \geq \rho\left((1-\alpha)E_1^{b_i} + \alpha E_2^{b_i}\right) + (1-\rho)\left((1-\alpha)E_2^{b_i} + \alpha E_1^{b_i}\right), \forall i = 1, \dots, l \qquad (15)$$

$$s.t:$$

$$E_1^{b_i} = \left(\frac{1}{2}\left(b_m + b_p\right)\right), E_2^{b_i} = \left(\frac{1}{2}\left(b_o + b_m\right)\right) \qquad (16)$$

In the above relationships, α is the uncertainty rate and ρ is the decision maker's view of the problem. If $\rho = 0$, the decision maker's view is optimistic, if $\rho = 0.5$, the decision maker's view is moderate, and if $\rho = 1$, the decision maker's view is pessimistic. According to the above equations, the controlled data coverage analysis model will be as follows:

$$Max\ \theta_0 = \sum_{j=1}^{J} v_j \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(b_{j0}^m + b_{j0}^p\right)\right) + \alpha\left(\frac{1}{2}\left(b_{j0}^o + b_{j0}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(b_{j0}^o + b_{j0}^m\right)\right) + \alpha\left(\frac{1}{2}\left(b_{j0}^m + b_{j0}^p\right)\right)\right) \end{bmatrix} - \sum_{k=1}^{K} w_k \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(c_{k0}^m + c_{k0}^p\right)\right) + \alpha\left(\frac{1}{2}\left(c_{k0}^o + c_{k0}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(c_{k0}^o + c_{k0}^m\right)\right) + \alpha\left(\frac{1}{2}\left(c_{k0}^m + c_{k0}^p\right)\right)\right) \end{bmatrix} \qquad (17)$$

$$s.t:$$

$$\sum_{i=1}^{I} u_i \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(a_{i0}^m + a_{i0}^p\right)\right) + \alpha\left(\frac{1}{2}\left(a_{i0}^o + a_{i0}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(a_{i0}^o + a_{i0}^m\right)\right) + \alpha\left(\frac{1}{2}\left(a_{i0}^m + a_{i0}^p\right)\right)\right) \end{bmatrix} = 1 \qquad (18)$$

$$\sum_{j=1}^{J} v_j \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(b_{jn}^m + b_{jn}^p\right)\right) + \alpha\left(\frac{1}{2}\left(b_{jn}^o + b_{jn}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(b_{jn}^o + b_{jn}^m\right)\right) + \alpha\left(\frac{1}{2}\left(b_{jn}^m + b_{jn}^p\right)\right)\right) \end{bmatrix} - \sum_{k=1}^{K} w_k \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(c_{kn}^m + c_{kn}^p\right)\right) + \alpha\left(\frac{1}{2}\left(c_{kn}^o + c_{kn}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(c_{kn}^o + c_{kn}^m\right)\right) + \alpha\left(\frac{1}{2}\left(c_{kn}^m + c_{kn}^p\right)\right)\right) \end{bmatrix} \qquad (19)$$

$$\leq \sum_{i=1}^{I} u_i \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(a_{in}^m + a_{in}^p\right)\right) + \alpha\left(\frac{1}{2}\left(a_{in}^o + a_{in}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(a_{in}^o + a_{in}^m\right)\right) + \alpha\left(\frac{1}{2}\left(a_{in}^m + a_{in}^p\right)\right)\right) \end{bmatrix}, \quad \forall n \in N$$

$$\sum_{j=1}^{J} v_j \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(b_{jn}^m + b_{jn}^p\right)\right) + \alpha\left(\frac{1}{2}\left(b_{jn}^o + b_{jn}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(b_{jn}^o + b_{jn}^m\right)\right) + \alpha\left(\frac{1}{2}\left(b_{jn}^m + b_{jn}^p\right)\right)\right) \end{bmatrix}$$
$$- \sum_{k=1}^{K} w_k \begin{bmatrix} \rho\left((1-\alpha)\left(\frac{1}{2}\left(c_{kn}^m + c_{kn}^p\right)\right) + \alpha\left(\frac{1}{2}\left(c_{kn}^o + c_{kn}^m\right)\right)\right) \\ +(1-\rho)\left((1-\alpha)\left(\frac{1}{2}\left(c_{kn}^o + c_{kn}^m\right)\right) + \alpha\left(\frac{1}{2}\left(c_{kn}^m + c_{kn}^p\right)\right)\right) \end{bmatrix} \geq 0, \quad \forall n \in N \tag{20}$$

$$v_j, w_k, u_i \geq 0 \tag{21}$$

4. Data Analysis

In this section, the implementation of data coverage analysis model on insurance companies in Iran based on the favorable and unfavorable inputs and outputs of Figure 1 has been discussed. The list of insurance companies considered for measuring return to scale is shown in Table 2.

Due to the uncertainty in the inputs and outputs of insurance companies and the lack of access to many historical data, the opinions of insurance industry experts have been used. Therefore, after collecting data from experts and averaging their opinions, the data has been classified into three categories: optimistic, probable and pessimistic. Inputs of insurance companies include three categories (total assets, share capital, and fee paid). Desirable outputs also include (net profit, reinsurance reserves, health and life, number of damages paid) and undesirable outputs also include (company debts, dissatisfaction and losses incurred). Each insurance company is considered as a DMU in the problem.

The model presented in the previous section has been solved using GAMS 24.2.3 software and CPLEX solver. Therefore, due to the use of the CPLEX solver, the presented results are accurate and have reached complete optimality. According to the analyzes done and the use of experts' opinions in conditions of uncertainty, when the experts answered the questions in an optimistic, probable or pessimistic way; Considering the uncertainty rate of 0.6, the input and output data are obtained as described in Tables 3-5.

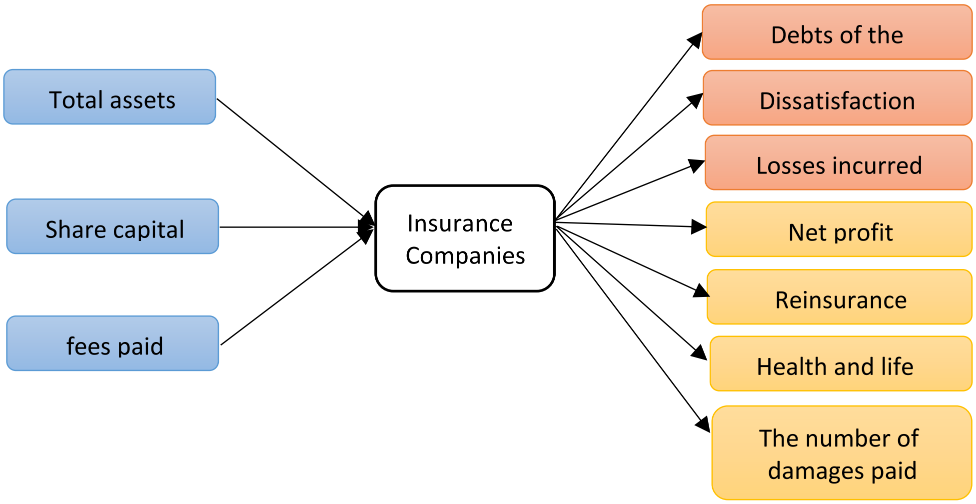

Figure 1. Inputs and outputs of the CCR model

Table 2. The list of insurance companies in Iran along with the main information

Row	Name of the Insurance Company	Number of Branches	Market Share	Response Time (Days)	Row	Name of the Insurance Company	Number of Branches	Market Share	Response Time (Days)
1	Taavon	38	0.8	2	13	Parsian	73	4.5	2
2	Middle East	1	0.1	5	14	Novin	65	1.8	2
3	Sina	63	2.5	3	15	Iran	212	31.9	3
4	Dana	86	10	2	16	Arman	37	0.6	36
5	Kausar	41	4.7	1	17	Ma	46	1.5	3
6	Alborz	61	4.9	2	18	Asmari	19	0.3	26
7	Razi	45	1.8	2	19	Asia	111	10.4	1
8	Moallem	62	5	2	20	Sarmad	37	1.6	2
9	Day	53	5.9	4	21	Mihan	43	0.6	8
10	Pasargad	88	2.7	2	22	Saman	35	1.6	3
11	Karafarin	56	1.5	2	23	Tejarat No	37	1	3
12	Mellat	14	1.9	3	24	Hekmat Saba	27	0.3	3

Table 3. Problem data for the optimistic case with an uncertainty rate of 0.6

Net Profit	Insurance	Total Assets	Share Capital	Fee Paid	Debts of the Company	Dissatisfaction	Losses Incurred	Net Profit	Reinsurance Reserves	Health and Life	The Number of Damages Paid
1.280	Taavon	3.120	5.600	5.520	5.600	6.240	3.900	1.280	3.760	3.200	2.640
1.760	Middle East	1.840	7.200	5.600	1.200	2.640	3.400	1.760	4.320	1.360	5.680
6.720	Sina	4.480	4.080	1.120	5.700	4.200	4.900	6.720	4.320	6.080	3.680
3.600	Dana	4.400	6.560	3.120	6.000	4.200	4.000	3.600	6.240	5.920	3.280
3.760	Kousar	4.240	4.400	2.880	6.500	2.280	5.900	3.760	6.880	6.000	6.000
1.840	Alborz	2.960	4.720	6.320	1.900	3.120	4.900	1.840	2.320	3.040	1.520
7.200	Razi	4.080	6.000	3.920	3.700	9.240	1.900	7.200	2.800	2.000	4.160
6.560	Moallem	6.800	5.760	1.200	6.900	2.880	3.800	6.560	7.120	5.200	1.600
5.280	Day	6.240	5.360	5.520	7.600	6.120	1.500	5.280	3.280	5.040	2.880
5.040	Pasargad	6.720	6.160	1.920	5.600	9.600	8.700	5.040	1.760	6.000	5.920
5.840	Karafarin	5.280	1.840	3.200	4.700	9.120	8.000	5.840	2.080	0.960	5.440
4.240	Mellat	2.000	0.960	6.160	5.600	4.440	2.300	4.240	2.480	4.560	3.280
4.960	Parsian	0.800	2.640	5.040	8.000	3.240	7.400	4.960	6.240	5.440	0.800
4.800	Novin	4.640	1.360	3.120	8.000	7.200	3.900	4.800	6.560	3.440	4.400
3.760	Iran	1.200	4.960	5.920	4.100	3.960	9.000	3.760	5.920	4.480	1.360
2.320	Arman	5.760	6.000	1.840	6.900	5.880	8.000	2.320	4.880	4.160	1.280
2.720	Ma	4.720	4.640	6.240	2.700	1.680	8.500	2.720	4.400	0.880	6.800
2.160	Asmari	4.000	6.080	6.400	7.400	4.920	7.700	2.160	4.160	3.200	2.080
7.120	Asia	1.440	1.440	2.800	2.800	1.920	7.900	7.120	3.120	1.760	2.480
4.320	Sarmad	4.320	5.280	2.960	8.100	4.080	4.700	4.320	6.000	2.800	4.720
1.600	Mihan	3.440	4.560	6.880	4.900	8.040	7.400	1.600	6.880	6.240	1.840
3.360	Saman	3.840	6.880	1.600	4.900	10.800	7.600	3.360	7.200	6.240	3.920
5.920	Tejarat No	5.840	5.520	7.040	1.000	7.920	3.200	5.920	1.840	5.840	5.280
4.640	Hekmat saba	5.840	2.240	3.760	1.200	7.440	2.100	4.640	6.560	5.840	6.160

Table 4. Problem data for the probable state with an uncertainty rate of 0.6

Insurance	Total Assets	Share Capital	Fee Paid	Debts of the Company	Dissatisfaction	Losses Incurred	Net Profit	Reinsurance Reserves	Health And Life	The Number of Damages Paid
Taavon	2.170	4.960	3.333	7.605	3.920	7.118	1.008	1.473	4.263	0.853
Middle East	5.425	6.433	5.580	3.608	5.145	7.898	5.348	3.643	5.270	2.480
Sina	3.488	5.193	3.798	6.435	10.168	5.363	6.588	0.775	3.875	2.093
Dana	1.783	5.503	5.658	5.460	6.248	4.583	3.178	1.938	6.045	5.115
Kousar	3.565	4.030	3.410	2.828	2.940	6.630	4.030	6.975	4.883	5.735
Alborz	4.108	2.558	6.278	5.753	2.450	7.703	5.503	3.100	3.100	1.395
Razi	1.938	0.853	3.720	5.753	9.188	7.898	3.333	2.403	1.860	4.883
Moallem	3.255	1.008	4.108	7.313	6.370	8.483	6.898	1.008	0.775	5.735
Day	6.820	6.200	4.340	3.900	2.205	2.633	5.193	5.348	4.573	3.953
Pasargad	6.200	3.953	1.008	7.995	7.595	3.218	5.813	2.325	2.480	3.953
Karafarin	4.573	3.643	2.480	7.605	2.450	6.533	5.968	4.960	0.853	5.890
Mellat	1.008	6.433	1.628	6.533	2.083	3.218	6.743	4.108	5.813	5.580
Parsian	6.898	3.953	3.875	3.705	10.413	5.265	5.813	5.348	3.023	4.030
Novin	1.550	1.628	6.200	2.438	2.818	4.680	5.425	1.395	5.735	4.650
Iran	2.015	1.938	4.883	1.170	3.798	5.168	2.558	4.728	1.628	0.930
Arman	1.860	6.820	1.395	8.580	9.433	6.825	5.115	6.510	0.930	2.248
Ma	4.650	6.278	6.355	6.143	4.778	7.020	3.178	3.488	2.248	4.340
Asmari	3.953	3.720	2.248	7.215	5.758	2.535	3.410	1.318	6.278	4.340
Asia	4.185	2.868	5.038	4.193	1.348	2.828	6.665	1.860	1.008	2.558
Sarmad	3.798	6.588	5.890	3.510	2.205	6.045	2.248	3.255	2.635	5.580
Mihan	2.325	0.853	1.163	7.995	5.513	5.265	5.425	3.100	0.930	5.813
Saman	6.123	0.853	6.898	7.118	9.433	1.560	1.395	5.425	2.403	4.340
Tejarat No	1.240	3.488	1.395	6.923	7.718	7.410	1.085	2.790	5.890	1.783
Hekmat saba	3.798	6.200	4.728	8.190	2.328	6.533	5.115	6.898	5.038	2.093

Table 5. Problem data for the pessimistic case with an uncertainty rate of 0.6

Insurance	Total Assets	Share Capital	Fee Paid	Debts of the Company	Dissatisfaction	Losses Incurred	Net Profit	Reinsurance Reserves	Health and Life	The Number of Damages Paid
Taavon	6.525	1.350	2.100	3.990	10.000	8.170	3.525	1.275	4.875	1.800
Middle East	5.400	6.675	2.925	1.330	9.875	4.750	1.125	4.425	1.875	3.600
Sina	6.150	4.725	2.400	7.695	2.625	1.045	5.025	4.800	2.550	1.575
Dana	2.325	5.775	5.775	1.425	2.875	3.420	1.800	6.675	6.075	4.950
Kousar	1.350	3.750	5.550	4.180	1.750	1.710	5.550	4.800	5.175	1.425
Alborz	2.175	5.700	5.625	3.990	7.750	7.885	3.825	0.900	0.900	5.400
Razi	4.875	5.100	2.100	5.035	9.125	7.220	4.350	1.275	4.500	1.200
Moallem	4.200	3.825	2.175	7.030	6.250	5.510	2.325	3.750	1.875	1.050
Day	1.575	2.925	1.875	1.140	9.000	5.225	4.200	1.650	4.500	5.925
Pasargad	6.750	2.175	5.250	4.560	7.875	4.845	3.975	5.025	5.100	4.275
Karafarin	3.075	6.150	6.750	5.605	10.375	2.660	3.375	2.250	4.800	2.100
Mellat	3.150	2.475	2.475	4.180	8.875	1.900	3.525	6.525	6.525	6.000
Parsian	3.750	2.625	5.475	7.410	9.000	4.750	5.175	1.950	0.825	3.225
Novin	6.450	6.525	2.025	5.415	8.500	3.895	6.000	6.225	3.150	1.350
Iran	5.700	3.975	3.150	4.275	10.000	1.235	6.375	4.350	1.125	3.000
Arman	4.050	0.900	4.650	6.840	5.625	8.170	6.525	4.800	3.300	3.750
Ma	4.800	1.800	2.475	5.130	9.500	3.040	1.500	4.650	2.025	4.125
Asmari	4.425	2.625	1.650	5.225	8.375	3.420	4.875	3.600	6.000	1.425
Asia	2.325	6.075	1.200	8.075	1.625	2.375	2.400	2.250	3.525	6.150
Sarmad	3.225	1.350	5.700	1.425	5.000	6.175	0.975	3.600	6.225	5.100
Mihan	1.575	3.000	6.075	7.410	4.375	5.130	2.100	2.775	4.950	1.950
Saman	2.925	2.700	4.725	1.805	7.000	7.980	6.150	6.525	3.675	2.175
Tejarat No	2.025	5.025	1.800	5.415	8.750	2.945	2.700	6.525	2.625	6.675
Hekmat saba	1.950	3.975	2.550	5.605	1.375	6.555	2.625	5.175	1.950	1.725

According to the analyzes made with GAMS software, the prioritization of insurances in three different perspectives under the conditions of uncertainty rate of 0.6 is as described in Table 6.

Table 6. Returns to scale of insurance companies in different perspectives

Name of the Insurance Company	Optimistically	Likely	Pessimistically
Taavon	1	1	0.907
Middle East	1	0.569	0.833
Sina	0.844	0.578	0.758
Dana	1	0.955	0.478
Kousar	1	0.819	0.645
Alborz	1	1	1
Razi	0.928	1	1
Moallem	1	0.822	1
Day	0.821	1	0.777
Pasargad	1	0.51	1
Karafarin	0.748	1	1
Mellat	1	0.622	0.966
Parsian	1	1	1
Novin	0.725	1	0.486
Iran	1	1	1
Arman	1	0.459	0.608
Ma	0.701	1	1
Asmari	0.594	1	0.431
Asia	1	1	1
Sarmad	1	1	1
Mihan	0.572	1	1
Saman	1	1	0.714
Tejarat No	0.6	1	1
Hekmat saba	1	0.939	0.404

The results of the above table show that the return to scale of insurance companies such as Iran, Asia, Parsian, Alborz and Sarmad always has a value of 1. As the expert view has shifted from optimistic to pessimistic, returns to full scale for insurance companies have declined. In this way, the average return to the scale of all insurance companies is equal to 0.8972 in the optimistic view, in the probable view it is equal to 0.8863 and in the pessimistic view it is equal to 0.8336. Also, the existence of uncertainty in the input and output data of the problem leads to a change in the yield to the scale of insurance companies. Figure 2 shows the average return to scale of insurance companies in different rates of uncertainty and in different views of experts.

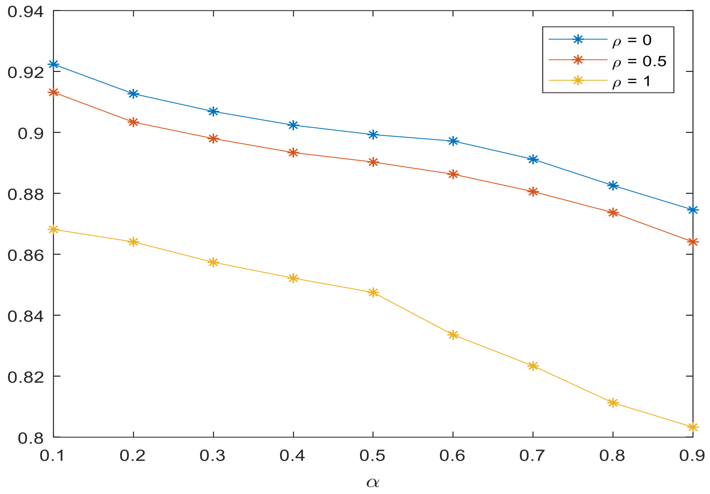

Figure 2. The average return to scale of Iranian insurance companies in different uncertainty rates

By examining the above results, it can be stated that the return to scale of insurance companies has decreased with the increase in the uncertainty rate in all three perspectives. This is due to increased uncertainty in inputs and outputs. So that with the increase of the uncertainty rate, the favorable outputs have decreased and the unfavorable outputs have increased, and this has led to a decrease in the efficiency of the scale of insurance companies.

Figure 3 also shows the return to scale of insurance companies in different perspectives and in the conditions of uncertainty rate of 0.6.

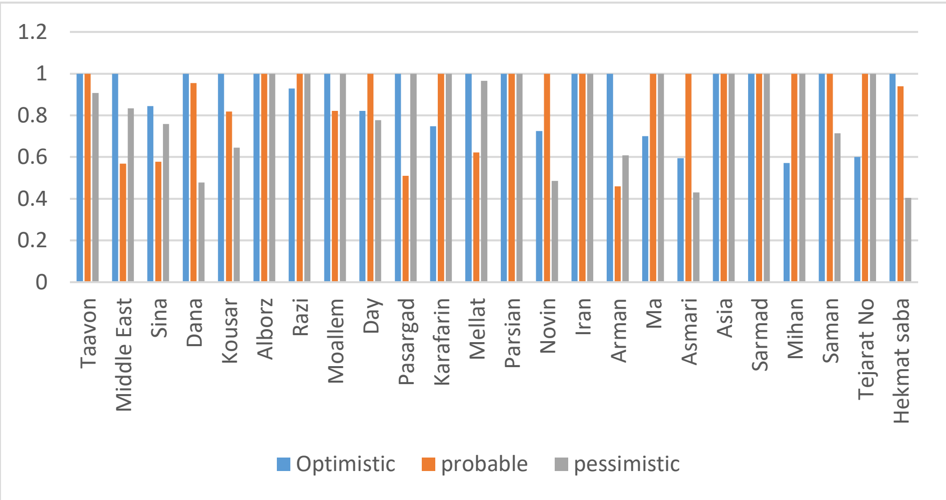

Figure 3. Return to scale of Iranian insurance companies at the uncertainty rate of 0.6

5. Conclusion and Presenting Suggestions

Insurance companies are known as one of the industries that influence the economy of any country. These companies are present in today's highly competitive market due to the protection of investors' capital in uncertain conditions. The proper efficiency of insurance companies will lead to investors investing in them with better confidence and leading to prosperity and economic growth of the country. Investigating returns to the scale of insurance companies is done with various methods, including data coverage analysis. In this method, the outputs of insurance companies are usually measured on their basis, and efficiency is obtained by increasing the outputs compared to the fixed inputs. In this article, taking into account two important principles, including the separation of outputs as desirable and undesirable outputs and considering uncertainty with fuzzy data based on different experts' views (optimistic, probable and pessimistic), a new model It was designed from the coverage analysis of the data to examine the return on the scale of Iran's insurance companies. In this model, the desirable outputs are separated from the undesirable outputs and leads to better conclusions regarding the return to scale of insurance companies. The designed model is in the condition of uncertainty so that with the increase of the uncertainty rate, the inputs of the insurance company and its desirable outputs decrease and its undesirable outputs increase. On the other hand, the views of the experts in the problem have an effect on the fuzzy data, so that the experts can have completely optimistic, probable or completely pessimistic views about the inputs and outputs of the model.

The calculation results on 24 insurance companies in Iran show that the average return to the scale of the entire insurance companies is higher in the optimistic view than in the probable and pessimistic view. That is, the more constructive the expert's view of the problem, the lower the return to the scale of insurance companies. On the other hand, the uncertainty rate also showed that with the increase of this value, the average return to the scale of insurance companies decreases. This is due to the reduction of the inputs of the problem, the desired outputs of the model and the increase of the undesirable outputs. For example, considering the uncertainty rate of 0.6, the return to scale of insurance companies such as Iran, Asia, Parsian, Alborz and Sarmad always has a value of 1. Also, the average return to the scale of the entire insurance companies is 0.8972 in the optimistic view, 0.8863 in the probable view, and 0.8336 in the pessimistic view. The results obtained from the model show that the managers of insurance companies can increase the return to the scale of their insurance company by increasing the desired outputs and trying to reduce the undesirable outputs. Also, the existence of different views of experts on the problem also helps the managers to get the most pessimistic and optimistic amount of return to the scale of their insurance company compared to all insurance companies.

The performance of insurance companies has been evaluated based on various indicators. However, there are many qualitative and quantitative indicators that cannot be presented in the mathematical model due to the lack of access to data. Therefore, it is suggested to use different indicators that affect the efficiency of insurance companies in future research. Also, due to the use of experts' opinions in obtaining the data of some indicators under the name of triangular fuzzy numbers, there is a possibility of differences in the results and evaluation of the efficiency of insurance companies. Therefore, the use of fuzzy robust method instead of fuzzy programming method in problem data control is proposed as another future research suggestion. This method has a higher efficiency than the fuzzy programming method in controlling non-deterministic data.

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